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MATHEMATICAL RESEARCH CONNECTED WITH OPERATION  
OF ELECTRONIC COMPUTERS

- USSR -

by A. A. Lyapunov

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└ This is a translation of an article written by  
A. A. Lyapunov in Matematika v SSSR za Sorok Let 1917-  
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In the last decade computational technology has become enriched with a new means of high capacity -- electronic computers. This has led to the fact that now it has become possible to obtain computational solutions of problems, which ten or fifteen years ago were quite impossible to solve. In this connection, a unique reestimate of values took place in the field of applied mathematics. Many mathematical methods, which have led to cumbersome computations, and which earlier were considered ineffective, now become quite workable. On the other hand, purely analytic constructions, which lead to inconvenient computational algorithms, have for a certain time lost their value in any case. In this connection many new problems have been advanced, connected with the development of methods for solving mathematical problems as applied to computers. These works are considered in the article "Approximate and Numerical Methods," and do not belong to the scope of the present article.

However, in addition, new problems arose in mathematics, the source of which was the operation of computing machines. These problems came into being because of the need of developing such methods of utilization of computers, at which their productivity becomes optimal, and also in connection with the desirability of solving with computers as wide a circle of problems as possible, without being confined to purely computational problems. Machines can perform a variety of logical operations, and if cleverly used they can facilitate the human intellectual labor in a great

variety of spheres. Finally, a large circle of interesting mathematical problems comes into being against the background of design of computational machines. All the foregoing questions pertain to the number of mathematical problems of cybernetics -- the mathematical science that has arisen in the last few years and which is engaged in the study of control systems and control processes.

Systematic scientific work in the field of cybernetics has developed in our country only in the last five years. It must be borne in mind that much that is of value in this field is performed by engineers, who are occupied in the design and operation of various control devices. Unfortunately, we did not have the capabilities of compiling a complete survey of all these works. We have therefore preferred to leave aside engineering works and confine ourselves to an examination of only mathematical works pertaining to the field of cybernetics. Here we have restricted ourselves only to work that is carried out in direct contact with electronic computers and which are mathematical in their profile. Such a limitation has led to the fact that essentially the present survey considers works performed by the cybernetics group of the Mathematics Institute imeni V. A. Steklov, Academy of Sciences USSR (MIAN) and at certain organizations working in contact with it.

The survey consists of four sections:

1. Theoretical investigations in the field of programming.
2. Non-arithmetic utilization of computers.
3. Theoretical investigation of control systems.
4. Certain other problems in mathematical cybernetics.

The third section is in close contact with certain sections of the article "Mathematical Logic and Foundations of Mathematics."

In October 1957 there was held at the Institute of Automation and Telemechanics Academy of Sciences USSR (IAT) a conference on relay-contact surface, at which S. V. Yablonskiy delivered a survey paper on the works of the MIAN in this direction. Section 3 of the present survey is to a considerable extent a reprint of this paper. In compilation of Section 2, great help was rendered by O. S. Kulagina

and T. N. Moloshnaya. Help in compilation of Section 3 was rendered by M. L. Tsetlin and O. B. Lupanov.

The title cybernetics covers many related problems, which are frequently studied independently, and which are nevertheless varied. The central concept of cybernetics is the concept of the control system. Furthermore, the particular contents of the mathematical problems concerning cybernetics lies in the study of the general properties of abstract control systems. Against this background, there naturally arises the further set of problems, namely the separation of classes of control systems, having certain properties, which are of importance in applications, and the study of the mutual relations between these classes, and also of methods of research and synthesis of control systems, pertaining to these classes.

S. V. Yablonskiy is responsible for an interesting attempt to give a rigorous definition of the concept of control systems. The definition proposed by him covers all the presently known control systems. Particular examples of control systems may be electronic and relay-contact circuits, the nervous system, the computational algorithm, the program, an algorithm for solving a certain problem that contains random tests, etc.

The definition of S. V. Yablonskiy takes into account the topology of systems, their functional features, and also the algorithm that controls their functioning. Unfortunately, the construction of such a definition is too complicated to quote in this article. In its meaning it has the same relation to real control systems as, according to Church's thesis, the abstract definition of the algorithm (recursion, Turing machine) has to the real algorithms used in mathematics.

### 1. Theoretical Research in the Field of Programming

In practical programming, great difficulty is produced by the fact that any complex program is constructed of a large number of individual parts, which interact in a quite complicated manner between each other. Therefore in direct programming it is necessary to imagine clearly the

structure of the entire program as a whole and its state at all stages of the work.

In this connection, the question arose of breaking up the process of programming into component parts so as to be able to compile, in an autonomous manner, programs for different parts of the problem, and then synthesize from them the entire program as a whole. Different approaches to the solution of this problem were developed in Moscow (A. P. Yershov, 1956 -- 1957, A. A. Lyapunov, 1952 -- 1953, R. I. Podlovchenko, 1956 -- 1957, and Yu. I. Yanov, 1955 -- 1956), in Leningrad (L. V. Kantorovich, L. T. Petrova, and V. A. Bulavskiy, 1955 -- 1957), and in Kiev (L. A. Kaluzhnin, 1956 -- 1957).

We shall detail briefly the contents of these works.

1. At the Mathematics Institute of the Academy of Sciences USSR in Moscow, a formalism was developed for logical program schemes (operator programming). At the present time this method is widely used in practice. It consists of the following: the algorithm of the solution of the problem is broken up into a series of elementary acts, each of which is sufficiently simple to be capable of being programmed directly. The sequence of performance of these acts may depend on various conditions. The part of the program which realizes each individual act of the algorithm is called the computation operator. The part of the program which realizes the verification of any condition on which the operating sequence of the operator depends, is called the logical condition (or the logical operator).

Frequently either type of operator depends on certain parameters, with different performance of the same operator taking place at different values of these parameters, and most frequently regular change of this parameter by unity corresponds to the successive performance of one in the same operator.

In order for the machine to be able to perform the alternating computation operator, it is necessary that the state of the memory be such as required by the operator.

In this connection special control operators are introduced. These are parts of the program, which prepare the state of the memory of the machine for the performance

of the computing operators. The entire program is synthesized of computing operators, control operators, and logical conditions.

Use is made of the following symbolism: the computing operator is denoted by capital Latin letters, and their dependence on the parameter is indicated by indices:

$A, B, C, \dots$ , etc. Logical conditions are denoted as variables of algebraic logic or as logical predicates by means of lower-case Latin letters:  $p, q, p(x > y)$ , etc.

The control operators are denoted also by capital Latin letters, and the brackets indicate of which of the functions of this operator consists. In most cases these are used in the programs control operators of certain standard types, and a standard symbolism is used to denote these operators. Thus,  $F(i)$  denotes an operator that increases the parameter  $i$  by unity wherever this parameter figures in the program. By  $O(i)$  is denoted an operator which restores the initial value of the parameter everywhere where it figures in the program. By  $\Phi(A)$  is denoted an operator that forms the operator  $A$  from the available ready subprograms.

A program scheme is called a line made up of operators and logical conditions, marked with arrows that indicate the sequence of performance of the operators, necessary for the realization of the programming algorithms, in the following sense. We give the name "terms of the scheme" to individual operators and logical conditions, its components. From each logical condition an arrow should go to one of the terms of the scheme or to its end.

The order of operations of the term of the scheme is determined in the following manner:

- 1) The farthest term of the scheme to the left is first performed.
- 2) Next, if the last of the performed terms was an operator, the term of the scheme standing to the right of it is performed.
- 3) If the last of the performed terms was a logical condition, then two cases are possible:
  - a) If the test condition was found to be true, then the term of the scheme standing directly to the right of it is performed.

b) If it is found to be false, then that term of the scheme is performed, to which the arrow points.

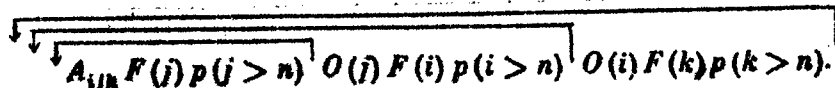
4) An empty place located after the extreme term of the scheme to the right, is considered as an operator, which stops the operation of the machine.

Let us give the following example of a program scheme.

Let the problem concern the multiplication of square matrices of order  $n$ :

$$\|a_{ij}\| \times \|b_{jk}\| = \|c_{ik}\|.$$

The first two matrices are written in certain cells of the memory. For the third matrix there are also earmarked definite memory cells. At the initial instant they contain zeroes. Let the operator  $A_{ijk}$  calculate the product  $a_{ij} \times b_{jk}$  and add it to the contents of the cell intended for the element  $c_{ik}$ . Then the scheme of the program can be constructed as follows:



The compilation of programs must be started with the development of a detailed scheme of the program. After the detailed scheme of the program is constructed, it is possible to program each operator individually. Here it is necessary to indicate the dependence of the operators on the parameters. Most frequently this dependence reduces to the fact that certain commands contain addresses, which depend linearly on the parameters. An exact indication of the dependence of the addresses on the parameters makes it possible to construct the control operators automatically.

G. P. Bagrinovskaya (1954) has shown that the construction of the controlling system of a program may be realized by the machine itself in accordance with a special programming program, provided the scheme of the program is given, the computing operators are programmed, and the dependence of the addresses of these operators on the parameters is indicated.

Later on M. R. Shura-Bura, E. Z. Lyubinskiy, S. S. Kamynin, and others have constructed programming programs, which construct fully the entire working program, on the

basis of the program scheme and a certain special information on the construction of the computation operators.

2. If the construction of the program is essentially dictated by the program scheme, so that realization of the programs in accordance with the ready scheme is performed automatically, then the question of the construction of the most suitable programs reduces to a considerable extent to being able to construct logical schemes that ensure convenient construction of the program.

It must be borne in mind that the requirements imposed on programs are quite different. They depend to a considerable extent on the machine and even on the operating condition. At the same time it is important to be able to select advantageous program schemes, without resorting to programming of the various versions that are being compared with each other. In this connection it is important to be able to transform the program schemes in an equivalent manner and to estimate the resultant program in accordance with the form of its logical scheme. The latter is not difficult to do in first approximation. It was found to be much more difficult to develop methods for formal transformation of program schemes.

This problem breaks up into two parts. The first of these consists of constructing such transformations, considering that the operators that comprise the programs remain unchanged, and only the form of the logical conditions are changed. Such transformations have been investigated in the work by Yu. I. Yanov.

The second consists of constructing transformations of program schemes with allowance for identical relations between operators. These transformations are intended for improvement in the construction of the control operators and the control system of the program. The first steps in the study of such transformations have been made in the works by R. I. Podlovchenko, N. Arent'yeva, and N. A. Krinitskiy.

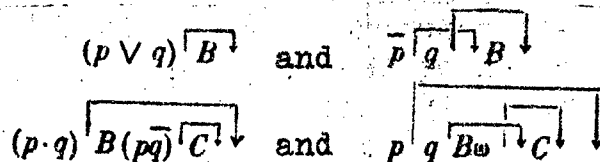
Let us give certain notions concerning the transformations of Yu. I. Yanov.

Assume that the logical conditions that are contained in the program scheme are functions of certain other logical variables. Then a verification of the initial logical



conditions can be replaced by a verification of new conditions, provided only the necessary transfers of control

are reconciled, for example  $(p \vee q) B \equiv p q B$ . These two schemes are equivalent to each other. Here is another example of equivalent pairs of schemes.



( $\omega$  denotes everywhere the identically false condition). Yu. I. Yanov has constructed a system of rules, which when used make it possible to transform into each other any two equivalent schemes of programs (in the absence of relations between the operators).

3) Another way of describing our algorithms was proposed by L. A. Kaluzhnin. The algorithm is broken up into elementary acts, which come in two types. One of these serves for the reprocessing of the information, and the other serves for definition of the further sequence of operation. Each operator, which realizes a certain act, capable of appearing in the algorithm, is represented by a point on a plane.

If the operator A must be followed by the operator B, then an arrow is drawn from the point A to the point B. If one of two given operators can follow a certain operator that verifies a certain condition, then arrows are drawn from the point corresponding to the first operator to both succeeding operators, and they are marked by the symbols "yes" or "no." The aggregate of all these arrows and notes, with indication of what operator represents the particular units, is called the graph of the algorithm scheme. L. A. Kaluzhnin and his students have studied methods of representing various algorithms in the form of graph-schemes, methods of constructing programs in accordance with the given graph-scheme with carrying this through to automatic programming, and also methods of formally transforming graph schemes of programs.

4. Still another approach to the question of formalization of the description of the structure of programs

is developed in the works by A. P. Yershov. The essence of the matter is as follows: one introduces a set of variables, a set of acts that convert the values of these variables, and a set of conditions that determine the sequence of operation of the elementary acts as functions of the values of the variables. The important fact is that the processed variables can serve in turn as codes for the transforming acts. The aggregate of the values of the variables, transforming acts, and logical conditions is called the computation algorithm. The principal difference between the construction of A. P. Yershov and the construction of L. A. Kaluzhnin is that the computational algorithm is capable of transforming the logical scheme proper, whereas the graph-scheme remains invariant during the operation, since all the possible logical paths are provided in it beforehand. From the point of view of the contents it is found that both the graph-schemes and the computational algorithms are equivalent to the generally accepted representation of algorithms in mathematical logic, i.e., if no limitations are imposed on the capacity of the memories, then both concepts are equivalent to the concepts of the Markov normal algorithm, the Church recursions, or the Turing machines.

The concept of the computational algorithm is so constructed that it represents in a very natural manner the construction of programs for computational machines, dispensing with the individual features of the machine.

Leaning on this concept, A. P. Yershov has developed a system of programming, which has been carried through by him to complete automatization. This system is realized in the programming program, which is distinguished by the fact that for programming problems in which the computations are of cyclic character, it requires merely that the counting scheme be specified, whereas the program scheme, which realizes these cyclical calculations, is selected by the programming program itself. Because of this, the program schemes to which this programming program leads are less varied than obtained by the programming program constructed by M. R. Shura-Bura, E. Z. Lyubinskiy, and S. S. Kamynin, but on the other hand the preparation of simple problems for programming with the aid of the A. P. Yershov program is

simpler. Further development of the theory of computational algorithms is quite promising.

5. The investigations of L. V. Kantorovich in the field of programming had a double purpose: on the one side, the question was raised of reducing the labor required for the compilation of the program, and on the other hand, there was a desire for expanding in a substantial manner the capabilities of the machines, by transferring to them not only the numerical counting, but also a considerable part of the analytical derivations or logical evaluations.

L. V. Kantorovich has succeeded in combining both types of capabilities in the programming system he developed, based on the use of special interpreting programs, which are called "prorabs." The idea of this method is as follows: The programming algorithm is represented in the form of a unique tree, the individual branches of which represent quantities that figure in the algorithm, while the nodes denote operations, which must be performed on the entering quantities in order to obtain the output quantity. This tree is coded by a special very compact method. The prorab explains what quantities are known and after observing that all the initial data for a certain operation are ready, constructs the program that realizes the necessary calculation. In the first versions of the prorab the operation of the machine was broken up in such a way, that the working program was worked out immediately, after which the control again was transferred to the prorab. In the new versions of the prorab, the working program can be retained and brought outside for independent utilization. The prorab should have ready sub-programs, with which it is possible to realize the operations that make up the algorithm. It must be noted that individual data can be not only numbers, but also vectors, matrices, or multi-dimensional matrices. The operations realized may be not only ordinary computational operations, but quite arbitrary operations, which we code the information. It is merely necessary that there be prepared sub-programs which realize these operations. Thus, for example, using the prorabs, it is possible to launch the machine on a solution of a differential equation with small parameter, using the expansion of the solution in

powers of the parameter, where the coefficients are obtained in the form of complex analytical expressions, containing indefinite integrals. These coefficients can be either tabulated, or given by the machine in analytic form.

The method of prorabs is particularly convenient in experimental counting, when it is necessary to be able to construct rapidly programs in which it is necessary to add a small number of times. Only the finely smooth variant of counting will be used to multiply. Under such conditions a certain additional expenditure in machine steps, unavoidable in utilization of the program, is not too dangerous. Apparently, the systematic utilization of the prorabs for the manufacture of sample calculations in the machines is very convenient.

6. In the Soviet Union, work is also being carried out on the practical programming and on the improvement of programming procedures, in particular on different methods of automatization of programming. In particular, a series of different versions of programming programs has been developed. Much work was also done pertaining to utilization of the method of standard programs. However, this work has not led to a formulation of independent mathematical problems. In the present survey we shall dwell only on those works in the field of programming, which already have led to theoretical investigations of mathematical character.

## 2. Non-Arithmetic Utilization of Computers

1. Theoretical investigations in the field of machine translation from one language to another. In principle, the computer is capable of realizing any conversion of information, provided one can describe the algorithm, according to which this transformation of information is to be performed. The only thing that limits the capability of the machine is the volume of the memory. However, in practice tremendous significance attaches to the carrying capacity that can be ensured by the machine in any particular case. It is very important to learn to ensure high efficiency of operation in a machine under as many types of

information transformation as possible. For this purpose it is necessary to pay particular attention to various experiments on the utilization of the machine on the transformation of information.

Particularly interesting is the performance of experiments of such a nature, in which the machine performs various functions of human intellect. Therefore an attractive feature is the possibility of transferring to the machine various types of utilization of human speech.

It is indeed from this point of view that translation of text from one language into another with the aid of computation machines is of principal interest.

Machine translation from foreign languages into Russian is being developed at the present time by a staff of workers of the Institute of Precision Mechanics and Computational Technology of the Academy of Sciences USSR (ITMiVT) and the staff of the MIAN, comprising O. S. Kulagina, T. M. Moloshina, G. P. Bagrinovskaya, jointly with I. A. Mel'chuk (Institute of Linguistics, Academy of Sciences USSR).

These staffs follow somewhat different ways. At the ITMiVT the principal tendency is the fastest realization along the path of the closest imitation of the action of the human translator. At the MIAN the principal tendency is a detailed study of the construction of translation algorithms and the development of methods that permit construction of such algorithms in as perfect a way as possible. In addition, attention at the MIAN to the development of a system of concepts, with which it is possible to represent grammar in the form convenient for machine and for needs of machine translation.

Both staffs carry out also work on automatization of the process of construction of translation programs.

At the ITMiVT work on machine translation from English into Russian began in the middle of 1954. In 1956 there were published the results of the first translation, obtained with the aid of the machine. At the present time work is being carried out on the construction of machine translation from German, Japanese, Chinese, into Russian.

At the MIAN, work on the construction of machine

translation from French into Russian began in the fall of 1954. In the summer of 1956 the first experimental translations were obtained. At the present time there is a theoretical study being carried out of the possibilities of machine translation based on material obtained with the experimental variant of translation from French, and an experimental variant of translation from English into Russian, based on the structural relationships between these languages, is being terminated. We now describe the work being carried out at the MIAN. The first version of translation from French into Russian was intended to translate mathematical texts with the aid of the machine.

On the one side, this makes it possible to confine oneself to a relatively small dictionary. On the other hand, this leads to a relatively simple structure of translated texts. At the first stages such a limitation of the problem was quite important, because it made it possible much earlier to proceed to the construction of translation programs and to experimental translations.

The principal work on the construction of rules of translation from French into Russian was carried out by O. S. Kulagina and I. A. Mel'chuk.

The first stage of the work consisted of the development of a system of rules, the formal application of which to French text would lead to a Russian translation of this text.

First these rules were formulated on the basis of observations on the process of translation. Correspondences were established between definite grammatical constructions of both languages. It soon became clear that the classification of words used in grammar is not always convenient in order to serve as the basis for machine translation. It became necessary to revise this classification. Thus, instead of the verb or adverb, the independent classes of words used had to be transitive or intransitive verbs, and the conjunctions are subdivided into coordinating and subordinate ones. Numerals and pronouns were regrouped and partially converted into nouns and adjectives.

The grammatical information sufficient for a strictly formal translation from French into Russian was gradually

outlined. This information breaks up into three parts. The first part is information on the individual words of the phrase, including their constant features, which are independent of the specific form of the word.

These are features of the grammatical basis of the words, which can be introduced directly into the dictionary. This includes such features as the part of speech, the type of formation of different forms of the word, methods of combination with other words, etc.

The second part of the grammatical information pertains to that form, in which the given word enters in the phrase. It is determined by the ending of the word. Tables of endings of all the variable parts of speech were compiled. A comparison of the endings of the analyzed word with these tables yields information on its grammatical form.

Finally, the third part of the information pertains to the structure of the phrase, and is extracted from an analysis of the mutual placement of the words in the phrase with the aid of a special algorithm, which makes up the analyzing portion of the translation rules. These rules verify the presence of various special constructions in the translated phrase and draw conclusions concerning the various Russian constructions with which the translation is to be realized.

After the necessary grammatical information on the construction of the Russian phrase has been obtained, the next problem arises -- that of constructing the Russian phrase. For this it is necessary to extract from the dictionary the Russian equivalence of French words, to place them in the necessary order, and to assign them the necessary grammatical forms. This makes up the second part of the rules, the synthesis.

Let us give several examples of phrases, translated with the aid of the "Strela" computer on the basis of the algorithm of translation developed at the MIAN.

1. Admettons donc que l'intégrale ait un certain nombre de points singuliers logarithmiques.

We admit thus that (in order that) the integral had a certain number of logarithmic singular points.

2. Indiquons une autre méthode pour établir

l'existence des intégrales des equations differentielles ordinaires.

We indicate another method in order to establish the existence of the integrals of ordinary differential equations.

3. Or, si le nombre  $k$  augmente indefiniment, on obtient  $A = x^n + x^{n-1} + \dots + 1$ , c'est-a-dire que tous les coefficients deviennent egaux a l' unite.

Thus, if the number  $k$  increases without limit, we obtain  $A = x^n + x^{n-1} + \dots + 1$ , that (in order that) all coefficients become equal to unity.

The programming of the translation represents great and specialized difficulties. On the one hand, the matter lies in the capacity of the program. Thus, a program of translation from French into Russian consists of 7,000 commands and 2,000 auxiliary constants (including the grammatical tables). It breaks up into 17 individual programs, which work sequentially. In the translation of each phrase all these programs work once one after the other.

The compilation and adjustment of such programs is a complicated and laborious matter. The specific difficulties are caused also by the fact that requirements are imposed to the translation programs of reducing the expenditure of machine time for experimental translations.

It was found advantageous to code the grammatical information differently at different stages of the work. Special cells are allotted to it and such methods are used for its coding, at which the number of cells used for storage is reduced. This information is extracted from the dictionary together with the words. This is followed by recording of the information.

A separation is made of those classes, in which are unified the words that influence in an equal manner the analysis and translation of the phrase. For each such class there is earmarked a special cell, in which a scale of a given class of words is constructed. The columns of the cell correspond to the words of the phrase. If a certain word belongs to the corresponding class of words, then in the column of this word is placed unity, and in the opposite case zero is placed there. Thus, the scale of a certain class of words reflects the distribution



over the phrase of the words of a given class.

Such a method of storing the working information makes it possible to verify in a unified manner whatever features that may be desired and requires a small number of machine steps for each individual verification.

The experimental translations performed made it possible to clarify what is the frequency of utilization of various operations in the work of translation programs and what requirements must be imposed at the present time on special translation machines.

The next step in the direction of general methods of constructing translation algorithms was the development of a machine translation from English into Russian (T. N. Moloshnaya). This algorithm was developed on the basis of principles which differ from the principles of French-Russian translation.

The role of the word in the English clause is determined not so much by morphology, as by syntactic means, i.e., by the sequence and grouping of the words in word combinations or by grammatical configurations. Therefore the rules for machine translation from English into Russian are based not on a word by word analysis of the clause, but on its analysis in accordance with the simplest syntactic connections, i.e., by elementary grammatical configurations.

The simplest grammatical configurations of the English and Russian languages were isolated and a correspondence between them was established. The translation was realized in the following manner: the English clause is subjected to analysis, which consists of finding the simplest grammatical configurations, and then the Russian clause is synthesized from the Russian configurations that correspond to the given English ones.

A certain difficulty in machine translation is produced by homonymy, a formal coincidence between several words, belonging to different parts of speech. For each case of homonymy, rules of distinction were developed. Part of these rules is based on grammatical endings, characteristic of definite parts of speech or classes of words. However, there are not enough such rules. These cases solve the problem whether a word belongs to a certain class of

words it is necessary to analyze the surrounding words. The program of translation from English into Russian is being developed at the MIAN by G. P. Bagrinovskaya and T. L. Gavrilova.

In these programs the grammatical information is also carried out on scales, and the analysis of configurations is carried out with the aid of logical operations, which are performed on these scales. As a result there are obtained special derivative scales for individual configurations first of English text, and then of the translating Russian text. It is regrettable that the structural trend in linguistics, which is quite fruitful for machine linguistics, is not being developed in our country at present as an independent scientific trend.

For further utilization of the data on the structure of languages in the interest of machine translation it is necessary to set up special investigations on linguistic statistics, particularly on the statistics of grammatical configurations.

The work done on the development of translation algorithms and their programming has led to the necessity of special theoretical investigations.

One problem lies in the construction of a certain system of set-theoretical concepts, with which it is possible to construct in a natural manner the grammar in any language. At the present time O. S. Kulagina has constructed a system, which is suitable for most European languages. The use of this system of concepts is convenient for the description of translation rules from certain European languages to others.

Another question lies in the development of a special system of elementary operators, from which it is convenient to synthesize programs of analysis of translation algorithms.

Such a system of operators was constructed by O. S. Kulagina on the basis of an examination of the programs of translation from the French.

These operators are adopted for the verification of logical conditions, and also for the recoding and transfer of individual elements of information from certain sections of the memory to others. A special compiling program, which

contains ready-made program parts for each of the indicated operators, is capable, after receiving the program scheme recorded with the aid of the foregoing operators, of transforming it into a corresponding translation program. This eliminates the principal portion of the labor in the work on compilation of translation programs. In recent times the circle of works that is carried out in the field of machine translation has been expanding.

I. A. Mel'chuk at the Institute of Linguistics, Academy of Sciences USSR, has developed an algorithm for the translation from Hungarian into Russian, essentially following the same path as developed for the translation from the French. He next undertook the development of principle of construction of an algorithm for the translation from any language into another one within the confines of a certain group. This algorithm consists of first explaining what grammatical means in each of these languages are used to express a certain relation between concepts expressed in words. Next, using formal grammatical properties, the relations are clarified between words in the translated phrase, after which a word by word translation is obtained, followed finally by a compilation, on the basis of information on the structure of the translated phrase, cast in a suitable grammatical form. These works are being carried out at present on the basis of French, Russian, English and Hungarian.

Closely related with machine translation are works on semantics and cybernetics, that are being carried out at the Laboratory of Electric Simulation (LE) under the leadership of V. A. Uspenskiy and V. V. Ivanov.

In Leningrad there is an all-city seminar on questions of machine translations, where work is beginning on the creation of translation algorithms for many foreign languages.

In Kiev there is also in operation a seminar on question of mathematical linguistics. A need is felt for establishing a closer contact between various staffs engaged in machine translation. This is connected with the fact that participating in this work are representatives of various specialties and different staffs. In this connection, many organizations (Moscow City Pedagogical Institute for

Foreign Languages, L.E, Institute of Linguistics, MIAN) have created a joint organization on problems of machine translation, the problems of which include the discussion of publication of works in the field of machine translation.

In Gor'kiy, under the leadership of V. A. Agrayev, there is a working group of linguists on the construction of translation algorithms from English and from French for radio physics texts. A dictionary of 200 words serves as a base.

2. The use of machines for economic purposes. Of great interest is the use of computers for various questions connected with control of the national economy. This includes such questions as the compilation of optimum plans under specified technical capabilities and specified requirements for the completeness of production, compilation of rational plans for transportation, finding advantageous prices for various types of products, and also the use of computers for planning technological processes and automatic control of manufacture. It must be noted that under the conditions of the planned socialist economy there arise unique problems, different from those economic problems which occur in capitalist economy. This difference is based on the fact that in capitalism the production is developed above all in the interest of the entrepreneur, whereas in socialism the production is developed on the basis of the interest of the socialist government and the nation. Abroad many persons are engaged in problems of mathematical economics, but essentially these researches are subordinate to the interest of private firms.

In our country work in the field of mathematical economy and the utilization of computers for economic purposes, which has reached the level of theoretical work of mathematical character, is carried out by L. V. Kantorovich and his co-workers, the mathematics group of the Laboratory for Control Machinery: A. L. Brudno, V. D. Belkin, A. G. Lunts, and others, and also by A. M. Gil'man in the Gor'kiy Physico-Technical Research Institute.

L. V. Kantorovich has proposed, as early as in the late thirties, a method of constructing optimal plans. This method consists of reducing the economic problems into a

a problem of finding the maximum of a linear form on a convex polyhedron in a certain multi-dimensional space. To solve such problems, L. V. Kantorovich developed a method of the so-called fastest descent. At the present time many various methods have been proposed for finding the extrema of linear functions on convex polyhedron.

Such problems are now called problems of linear programming. (Here the term of programming is connected with the fact that one is usually interested in a rational program of action. It has no relation to the "programming" for computers.)

All these methods require, in cases of practical importance, the performance of very laborious computational work. They therefore require the use of computers.

The works of A. L. Brudno and his co-workers have as a purpose the construction of apparatus, with which it would be possible to determine the outlays for manufacture of a definite product and to find rational costs. The method consists of investigating the matrix of "interactions" of several branches of industry. Each row corresponds to a definite finished product, and each column to a definite product used in the manufacture. At the intersection are placed the quantities of the consumed product, necessary for the manufacture of a unit finished product, and its tentative costs. A separate column is earmarked for labor costs. For the products bought on the outside market, a separate column is also allotted. The iteration of such matrices makes possible a determination of the total expenditures of certain products for the manufacture of others as well as rational prices.

Analogous methods have been worked out by V. Leontev in the U.S.A. as applied to the economic-capitalist society.

Quite recently A. G. Lunts observed that an examination of similar concepts was made as early as in the sixties by a Moscow professor of economics, Dmitriev.

It was found that the results of Leontev and Dmitriev, although they do not agree with each other, owing to a certain difference in the initial concepts, they can be readily reduced one into another,

These works are of great interest for the development

of mathematical economics of a socialist society. On the other hand, it is very important to develop work on a direct control of technological processes. Here notice should be taken of the work of A. M. Gil'man who has developed a procedure for designing the technology for lathes with the aid of a computer.

3. Games in machines. The playing of intellectual games with machines is of significance in the establishment of ideas concerning the capabilities of the machines, and also for finding general principles of increasing the productivity of algorithms.

In the realization of any game in a machine the question arises of the formalization of the tactics, rational realization of the tactical algorithm, and the coding of the gain information and the rules of the game.

Recently the following games have been incorporated in machines: V. M. Kurochkin has constructed a program for solving two and three move chess games with the aid of a BESM machine. A. N. Krinitskiy has constructed a program for the game of "nim," Yu. A. Derving for a game in "goat" dominoes, and N. A. Guskov and A. N. Lyapunov for certain versions of tic tac toe. These programs have been constructed for the "Strela" computer.

All these programs for the realization of games by machines are quite successful.

### 3. Theoretical Investigation of Control Systems

The question of the synthesis of control systems has two aspects: technical and mathematical. The technical aspect is connected with the construction of specific devices. The technical topics have given rise to the mathematical ones. A great role has been played in these questions by the works of our compatriots V. A. Shestakov, M. A. Gavrilov and others, and also by the foreign scientists Shanon, von Neumann, Aiken, and others.

At the present time there are in the Soviet Union many staffs that are developing methods of synthesis of control devices. A considerable part of these investigations

is essentially of engineering character. However, this survey will consider only those works on the synthesis of control devices, in which the mathematical aspect is predominant.

In the development of the theory of control systems one can note two tendencies. The first consists of studying methods of synthesis of controlling devices from a greater and greater variety of elements. This includes relay-contact electronic, ferrite, transistor, and other circuits. On the other side, the question arises of investigating the construction of the principles of synthesis of control systems made of elements of definite but in general arbitrary nature, which is connected in a natural manner with the establishment of a general and exactly determined concept of the control system.

The second trend in the investigation consists of solving several mathematical problems which have arisen in the theory of contact circuits, but which have quite general significance and for a long time have not been amenable to solution, owing to the mathematical difficulties.

In both these trends, substantial results were obtained in the Soviet Union during the last decade.

The process of expanding the field of research takes place not only through simple expansion of the set of investigated objects, but also by including into a single consideration the regions which have already been developed (for example, the theory of graphs, disjunctive normal forms, etc.). With this, a comparison takes place of objects or statements of problems, and this frequently leads to the use of results from one region in another one, and sometimes even to a carry over of theorems and methods.

Furthermore, inasmuch as the same problems arise for various schematized objects, it becomes possible to select a "model" object, which is less encumbered with secondary peculiarities that make the solution of the problem less difficult than others. The success of solving a problem depends to a considerable extent on the choice of the model object. For example, it was found that to ascertain the difficulties in the synthesis of networks it is simpler to consider oriented networks (of the electronic type) than

contact networks (which are essentially not oriented). It is possible that an attempt of establishing asymptotic laws on the basis of contact circuits has generally retarded the searches for asymptotic laws for networks.

In recent years there has arisen and there was rapidly developed a new branch of cybernetics -- the theory of automata, the ground work for which was laid by the works of McCullough and Pitts, Kleene, Berns, Wright, J. U. Neumann, Moora, Culbertson, and others.

Essentially this abstract examination of the work of the computing machine with allowance for the fact that its memory has a strictly volume. Each automaton is a net, having a definite entrance and exit, the entrance being subject to external influences. The time is considered discrete. At the nodes of the net are placed the elements which are called neurons, capable of converting in a definite manner the incoming information. Each such element can have many inputs and outputs. The signals that circulate in the automaton are capable of assuming one of two values, the same as the element of the nervous system, the neurons, are capable of assuming one of two possible states. Depending on the state in which the element is found, it either delivers or does not deliver a pulse at the output. Depending on what pulses are received, the element at the next instant of time will assume a particular state.

The principal problem consists of finding at what are the sets of external stimuli which can be recognized by the automaton, and also the question of the method of synthesis of automaton. The question of what external events can be presented to the automaton were solved by S. Kleene.

The principal problem consists of finding at what are the sets of external stimuli which can be recognized by the automaton, and also the question of the method of synthesis of automaton. The question of what external events can be presented to the automaton were solved by S. Kleene. Another solution of this problem, which simplifies and develops the results of Kleene, was obtained by Yu. T. Medvedev. L. S. Skornyakov applied algebraic methods to the study of automaton.

Let, for example, there be a device (Fig. 1) with



discrete  $n$  input channels and  $m$  discrete output channels, which can be considered as a device with one generalized input and one generalized output channel (Fig. 2).

It is assumed that all the channels can be found in a finite number of states, and the state  $\beta_n$  of the output at the instant of time  $t_{p+1}$  is determined fully by the states  $\alpha_1, \alpha_2, \dots, \alpha_n$  of the input at the instants of time  $t_1, \dots, t_p$ .

The device (Fig. 2) converts the sequence  $\alpha = \alpha_1, \alpha_2, \dots$  into a sequence  $\beta = \beta_1, \beta_2, \dots$ . This conversion defines a certain operator  $T$  such that

$$T\alpha = \beta.$$

The following question arises: in what manner does the nature of the operator  $T$  influence the simplicity of the circuit. To investigate operators of this kind, B. A. Trakhtenbrot proposed a graphic method (the method of trees) and found the characteristic of the operator  $T$ , called the weight, and estimating its complexity, B. A. Trakhtenbrot established the properties of operators with specified weights and showed that the magnitude of the weight determines the number of delay elements, i.e., the capacity of the internal memory of the automaton.

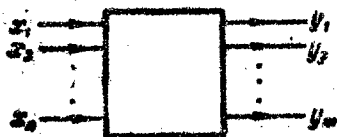


Fig. 1



Fig. 2

It is known that the choice of language with which the information processing is described (tables, graphs, functions of algebraic logic, sequences of functions of algebraic logic, matrices, etc.) influences the procedure of the circuit synthesis, particularly for individual classes of functions. The question therefore arises of finding a new means for describing the operation of the circuits. Incidentally sometime it is required to construct on the basis of this description the canonical equations for the operation of the automaton, determined

by the relations

$$y(t) = \Phi(x(t), z(t-1)), \quad z(t) = \Psi(x(t), z(t-1)),$$

where  $x$ ,  $y$ , and  $z$  are respectively the variables that describe the input and output channels and the internal memory (Fig. 3).

M. L. Tsetlin introduced for the description of the operator of such circuits special matrices on Boolean variables, which permit in many cases to synthesize the circuits directly. B. A. Trakhtenbrot proposed to use for the recording of the information processing an expanded calculus of predicates, in which quantors by predicates are permitted, as well as limited quantors by object predicates and only single-place predicates. There are also many other investigations that yield different approaches to the solution of the problem of synthesis of automata. Thus, M. L. Tsetlin, on the basis of a special matrix calculus, considers the question of the composition of automata and on their breakdown into semi-automata.

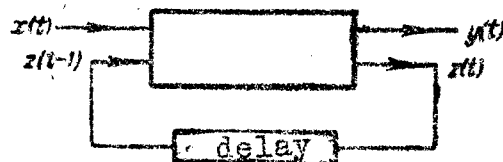


Fig. 3

An interesting language for the description of the operation of automata was proposed by Yu. Ya. Bazilevskiy.

In addition to the works mentioned above, pertaining to the synthesis of networks independent of the physical nature of the component elements, there have been obtained in recent years considerable results in the field of synthesis of networks made up of concrete elements.

Thus, for example, there has been an extensive use of tabular (more accurately, graphic) method of synthesis of so-called multiple-step relay circuits, expounded in the well-known monograph by M. A. Gavrilov and then developed in the works of V. N. Roginskiy, V. G. Lazarev, and others. In this procedure the table (graphic) of connections shows the sequence of states of the inputs of the network and of its output. Next, with the aid of special

devices (satisfaction of the requirement of non-contradiction) this table is supplemented by sequences of states of intermediate relays. The network can be constructed in accordance with the fully compiled table.

V. I. Testakov proposed a vector-algebraic method of synthesis of so-called autonomous relay circuits, which made it possible for him to formulate certain general premises. This method was generalized by V. I. Shestakov to include non-autonomous networks.

On the basis of research performed by S. V. Yablonskiy on k-valued logics there were developed from the function theoretical point of view methods of synthesis of electronic circuits, using multiple-valued logics.

M. Ya. Eyngorin gave a method of constructing memory devices for a given number of states, using less tubes than ordinarily. Using already the above-mentioned matrix method of synthesis of networks, M. L. Tsetlin proposed an algebraic formalism for the synthesis with triggers. M. L. Tsetlin and L. M. Shekhtman developed an algebraic formalism for the synthesis of controlling devices made up of single-type ferrotransistor cells.

A considerable number of works was devoted recently to searches for effective methods of construction of contact circuits. In this field, the work by M. A. Gavrilov, devoted to so-called bridge-circuits, have received great renown. The effective general method of synthesis of contact networks (the so-called cascade method) was proposed and investigated in detail by G. N. Povarov.

An important role was also played by methods of taking into account the so-called unused states, developed by V. N. Roginskiy, who proposed also a suitable algebraic formalism (transformation of equivalences).

The mathematical aspect of the problems arising here was considered in detail in the paper by Yu. I. Zhuravlev.

The method of synthesis of contact networks, using equations of algebraic logic, was proposed by V. N. Grebenshchikov.

Recently there has been a considerable development of matrix methods of investigation of contact networks, proposed by B. I. Aranovich and A. G. Lunts. The elements of the

matrices introduced in these methods are given by the admittance between the poles of the contact multi-pole network. Next A. G. Lunts develops a complete algebraic formalism, which makes it possible to perform various transformations over networks of the most general type, to simplify networks, etc. The methods of A. G. Lunts were further developed in the works by G. N. Povarov and in many foreign papers. We note incidentally that the matrix method was used by G. N. Povarov to solve many interesting logical problems.

Another matrix method of the synthesis of contact networks was proposed by M. L. Tsetlin. The essence of this method consists of segregating in the network the input and output buses and constructing rectangular matrices, the elements of which are the admittances between the input and the output buses. The method is particularly effective in the synthesis of network consisting of individual blocks.

A. Sh. Blokh is responsible for a further consideration development of this method, which reduces in general outlines to the introduction of an intermediate integer parameter for the matrix functions.

A. V. Kuznetsov and B. A. Trakhtenbrot have developed topological methods of the theory of contact networks. A. V. Kuznetsov has proved that for non-planar networks there exists no analogue whatever of the duality principle, which would retain the same number of contacts. He also found conditions of non-separability of non-repeated contact networks (i.e., networks, having only one contact on each relay). B. A. Trakhtenbrot obtained the conditions for the possibility of realization of a function by means of a non-repeated network and developed a method of synthesis of such networks, and also showed essentially the uniqueness of such a realization.

Many works have been devoted to the synthesis of networks that realize individual classes of functions of algebraic logic. We note here the works of G. N. Povarov, V. K. Korobkov, B. I. Finikov, and others. An interesting and important from the point of view of application is the call of so-called commutation networks, which was investigated by A. K. Kharkevich.

An important circle of problems, which arise in the theory of contact networks, includes the construction of general methods of synthesis of networks with bridges. These problems reduce to a study of general properties of aggregates of functions of algebraic logic and other objects. These problems continue the classical works of C. Shannon and were considerably developed in the most recent years. Let us dwell on these problems in somewhat greater detail.

To estimate the complexity of the system of networks, Shannon introduced a function  $L(n)$  which represents for each  $n$  the smallest integer such that each function that depends on  $n$  arguments can be realized by a network with a number of elements not exceeding  $L(n)$ .

It is obvious that different synthesis methods give a possibility of estimating  $L(n)$ , and from the divergence between the highest and lowest estimates for  $L(n)$  we can judge the value of a particular method.

Already in 1949 Shannon has shown that

$$(1-\varepsilon) \frac{2^n}{n} < L(n) < (1+\varepsilon) \frac{2^{n+2}}{n}.$$

These estimates diverge by a factor of four and the question of the relative behavior of  $L(n)$  at large values of  $n$  remains open. Many attempts were undertaken in this direction. An effective method of synthesis (so-called method of cascades) was obtained in this direction by G. N. Povarov, who improved the Shannon method. However, from the asymptotic point of view the method of G. N. Povarov yielded nothing new. Quite recently O. B. Lupanov succeeded in showing that  $L(n) \sim \frac{2^n}{n}$ . This solved Shannon's problem. The latter was accomplished by constructing a new method of synthesis, using refined considerations with respect to the construction of functions of algebraic logic. As an intermediate result, O. B. Lupanov gave the construction of a multi-pole network, which realizes all the constituents of unity in the variables  $x_1, x_2, \dots, x_n$ , and which contains asymptotically one half the number of contacts contained in a tree.

O. B. Lupanov investigated also methods of synthesis (with estimates) of networks made up of so-called functional elements, contact-valve, parallel-series

networks and a few other networks, and in many cases succeeded in obtaining asymptotic expressions.

R. Ye. Krichevskiy considered formulas which were superpositions of certain basic functions. He obtained a result which when applied to  $k$ -valued logic reads as follows: almost all the functions of  $k$ -valued logic  $f(x_1, \dots, x_n)$  require, when written down in terms of basic functions, asymptotically not less than  $C \frac{k^n}{\log n}$  letters of variables

(generalization of the Riordan-Shannon theorem).

The preceding results show that the overwhelming part of the functions require for realization an order of  $\frac{2^n}{n}$  elements (contacts, contacts and valves, etc.),

i.e., has the most complicated circuits. Essentially, they show that it is not advisable to raise the question of working out single methods of synthesis for all functions.

Along with these question, of great interest is the attempt to prove the minimality of various schemes. This is quite a difficult question.

C. Cardo established a minimality of circuits that realized linear functions.

A. A. Markov obtained a method of synthesis of minimal contact-valve networks for monotomic symmetrical functions. This method makes it possible to construct minimal networks with a number of contacts on the order of the square of the number of variables (the methods hitherto known admitted only of the construction in specific circuits with linearly increasing number of contacts).

A. A. Markov investigated also the interesting question of the inversion complexity of functions of Boolean algebra.

In order to ascertain the sufficient conditions for minimality of a network, Yu. L. Vasil'yev reviewed a large number of networks. He compiled a catalog for networks that realize functions of four variables. The basis for the catalog was the catalog prepared by G. N. Povarov and the catalog of Igonnier and Gray.

Mention must be made also here of the numerous works devoted to the creation of machines for automatic synthesis of networks performed by T. T. Chukanov,

V. N. Rodin, P. P. Parkhomenko, A. A. Arkhangel'skaya, V. G. Lazarev, and V. N. Roginskiy, and others.

An interesting problem, connected with network theory, was considered by S. V. Yablonskiy and I. A. Chegiz, who created the general theory of construction of tests for detection of faults in networks in the case when these faults have a systematic (not random) character and the only means in the "ringing" of the network, i.e., analysis of the network operation under definite external influences.

It was found here that there exists a direct relationship between the construction of tests and the construction of so-called blind-alley disjunctive normal forms.

Let us dwell also on the work of Yu. I. Zhuravlev, in which he considered the question of the choice of selection from a table. The operation, admissible for the choice, consists of verifying individual elements of the table. Yu. I. Zhuravlev solved the problem of construction of an algorithm for solving a problem of choice with the aid of asymptotic minimal number of acts.

Very important results, belonging to S. V. Yablonskiy and connected with network theory, will be described in the next section.

#### 4. Certain Other Problems in Mathematical Cybernetics

In this section we discuss briefly important results obtained in the most recent time and which pertain to many mathematical problems that arise in cybernetics.

In questions pertaining to the synthesis of contact networks there arise unexpectedly certain circumstances of set-theoretical nature.

In the theory of sets it is frequently necessary to distinguish constructions which are effective in a certain sense, and constructions that involve the arbitrary-choice axiom (the Zermelo Axiom). In the latter case the constructions contain, as a rule, a set of elementary acts of considerably greater potency. The synthesis of minimal contact network, that realizes the special function of algebraic

logic, can always be performed by a finite number of steps. However, in the general case this requires the scanning through all possibilities within a definite limit, i.e., for the synthesis of minimal networks, for a definite function, it is necessary essentially to scan all such networks, which can be constructed from the necessary number of elements.

This naturally gives rise to the question of how to make the synthesis close to minimal by using a substantially smaller number of elementary acts. This approach has much in common with the so-called effective approach to the theory of sets.

Important results in this direction were obtained by S. V. Yablonskiy in the most recent time. He examined the question of the construction of such classes of functions, for which it is advantageous to create a special algorithm for the synthesis of the networks that realize them.

He introduces classes of functions of algebraic logic, called invariant. These classes are characterized by the fact that they are invariant under renaming of the variables and substitution of constants, and also relative to the inclusion of non-essential variables.

Let  $Q$  be a certain class of the type described. We denote by  $P_Q(n)$  the number of functions that depend on the variables  $x_1, \dots, x_n$  and enter into the class  $Q$ .

Let us examine the expression  $\sqrt[n]{P_Q(n)}$ . It is found that it only decreases with increasing  $n$ . Let us put

$$\lim_{n \rightarrow \infty} \sqrt[n]{P_Q(n)} = 2^\sigma,$$

where  $0 \leq \sigma \leq 1$ . Let  $L_Q(n)$  be the number of contacts, with which it is possible to realize any function of class  $Q$ , which depends on not more than  $n$  variables. Then if  $\sigma > 0$ , then a method of synthesis is indicated at which

$$\lim_{n \rightarrow \infty} \frac{L_Q(n)}{L(n)} = \sigma.$$

We note that  $\sigma \neq 1$  there is a continuum of classes, all different, satisfying the conditions of S. V. Yablonskiy.



At  $\sigma=1$  these conditions are satisfied only by the class of all functions of algebraic logic.

The method of synthesis of functions that enter into a certain class  $Q$  requires a substantially smaller scanning of possibilities than the method of synthesis that is suitable for all functions of algebraic logic.

Another question considered by S. V. Yablonskiy in the same paper, consists of the following: let there be given a certain sequence of functions of algebraic logic. It is asked what can be said concerning the smallest invariant class, which contains this sequence. It is found that if such a sequence is formed by a random choice such that at the  $n$ -th step there can be chosen with equal probability any of the variables  $x_1, \dots, x_n$ , then with probability 1 the foregoing invariant class coincides with all the functions of algebraic logic. However, to establish the fact that a certain individual sequence of the function gives rise in the above-mentioned sense the class of all the functions of algebraic logic, it is necessary again to review all the possibilities. The situation obtained here is such that the Zermelo approach can be replaced with probability 1 by the Monte Carlo method, but the inclusion of the "unfavorable" event of 0 probability again requires a complete review of all the possibilities.

The foregoing work of S. V. Yablonskiy is of great interest from the point of view of following the role of general principles of theory of sets in mathematical problems of cybernetics. In this work there comes to the forefront the question of the mutual relation between the objective difficulty, contained in the solved problem and the complexity of the algorithms, with which the given problem is solved.

A. G. Vitushkin approached the question of algorithms with estimates in a somewhat different manner. He started out with the study of multi-dimensional variations of functions of many variables and the images of functions of many variables of a given degree of smoothness with the aid of superposition of functions of a lesser number of variables of specified smoothness. A. N. Kolmogorov observed that these questions lead to an examination of the information contained in the  $n$ -approximation of an arbitrary function of a given class.

In this connection, A. G. Vitushkin raised the question of how to estimate the difficulty of calculating an arbitrary function of a given class, if it is known how much information it is necessary to specify in order to obtain the  $\epsilon$ -approximation to any function of this class.

The difficulty of the problem of calculating the individual function is made up of two elements. Let there be a class of algorithms which make it possible to calculate any of the investigated functions.

These algorithms are characterized by a certain general structure and differ from each other by the numerical values of certain parameters. To fix the individual function it is necessary to specify the values of these parameters. In addition, to calculate the values of the function from the specified values of the argument it is necessary to carry out a definite number of operations in accordance with a selected algorithm.

Thus, in order to judge the "quality" of the algorithm we have two parameters:  $p$  -- the number of parameters on which the individual algorithm depends and  $k$  -- the number of operations performed by this algorithm.

A. G. Vitushkin established that if the smoothness of the class of function is determined by their differential properties and the number of arguments, on which they depend, then if we denote by  $H_\epsilon$  the  $\epsilon$ -entropy of a certain class of functions, then for any class of algorithms, which give a possibility of calculating any of these functions with accuracy to  $\epsilon$ , the following inequality holds:

$$p \log(k+1) \geq H_\epsilon.$$

The quantity  $H_\epsilon$  can be determined for many classes of functions from formulas given by A. G. Vitushkin in an earlier paper  $H_\epsilon = C \cdot \epsilon^{-\frac{n}{q+\alpha}}$ , if one considers the class of all functions of  $n$  variables, which admit of  $q$  derivatives in all the variables, whereas the latter derivatives satisfy the Holder condition with exponent  $\alpha$ . The papers of A. G. Vitushkin on the study of algorithms of tabulation, which are described here very incompletely, are distinguished for the great depth of their results and the force of the function-theoretical construction.

Like the works of S. V. Yablonskiy, these pertain to the general problem of clarifying the connection between the internal structures of objects and the methods of their formation. In this sense, they form an intermediate link between the theory of functions and cybernetics.

The works described represent the first steps in the field of mathematical problems in cybernetics. They are unified by a certain general direction of thought, which can be characterized at the start of the development of general metric theory of algorithm or theory of algorithms with estimates. However, the construction of such a theory is still a matter for the future. This field is closely adjacent to mathematical logic and the theory of algorithms, and also to general ideas of set theory and theory of functions of real variables. The statement of the problems in this region is closely related to many problems that are already outside of mathematics. This, on the one hand, is control engineering, and on the other hand these are the biological sciences, primarily physiology, genetics and evolution biology, and, finally, economics. The development of this field is of great significance for the expansion of the spheres of application of mathematical ideas and to a great variety of fields of human activity. This expansion of spheres of application of mathematics is one of the most characteristic general phenomena of science in the middle of the 20th century.

END